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SOLUTIONS OF PROBLEMS IN NUMBER ONE.

Solutions of problems in No. 1, have been received as follows:—

From R. J. Adcock, 386; Marcus Baker, 376; Alex. S. Christie, 375, 377, 379, 382; Prof. L. G. Barbour, 386; Prof. W. P Casey, 375, 376, 377, 279, 381; Prof. H. T. Eddy, 384, 385, 386; Dr. Eggers, 376; Prof. E. J. Edmunds, 375, 377, 380; George Eastwood, 386; H. B. Goodnow, 384; Prof. A. Hall, 384; H. Heaton, 376, 380, 382, 383, 384, 385; T. N. Haun, 375; W. E. Heal, 379, 380, 383, 384; Dr. A. B. Nelson, 375, 376, 377, 380, 381, 382; Paul Peltier, 379; Prof. E. B. Seitz, 375, 376, 377, 379, 380, 382, 383; Prof. J. Scheffer, 375, 376, 377, 380, 383; Prof. D. V. Wood, 380; E. Vansickel, Jr., 375; Alexander Ziwet, 379.

375. *By W. B. Bates.*—"*A* and *B* enter into partnership and gain $200. Now six times *A*'s accumulated stock (capital and profit) equals five times *B*'s original stock, and six times *B*'s profit exceeds *A*'s original stock by $200. Required the original stock of each."

SOLUTION BY T. N. HAUN, GREENVILLE, E. TENNESSEE.

Let x = A's original stock. Then, by conditions of the problem.

$$\tfrac{1}{6}(x+200) = \text{B's gain};$$

$$\therefore\ 200 - \tfrac{1}{6}(x+200) = \tfrac{1}{6}(1000-x) = \text{A's gain, and}$$

$$6[x+\tfrac{1}{6}(1000-x)]\div 5 = x+200 \quad = \text{B's original stock.}$$

Since their gains are proportional to their original stock, we have

$$x : x+200 :: \tfrac{1}{6}(1000-x) : \tfrac{1}{6}(x+200),$$

or

$$x : x+200 :: 1000-x : x+200;$$

$$\therefore (x+200)x = (1000-x)(x+200).$$

Whence we get $x = \$500$, A's stock, and

$$500+200 = \$700\text{, B's stock.}$$

376. *By Dr. Eggers.*—"Divide a right angle into three parts, such that the tangents of the several angles are proportional to three given numbers."

SOLUTION BY PROF. J. SCHEFFER.

Denoting two of the angles by α and β, and the given ratio by $m : n : p$, we obtain from the proportions

$$\tan\alpha : \tan\beta \quad = m : n,$$

$$\tan\alpha : \cot(\alpha+\beta) = m : p,$$

$$\tan \alpha = \frac{m}{\sqrt{(mn+mp+np)}}, \ \tan \beta = \frac{n}{\sqrt{(mn+mp+np)}},$$

$$\tan \gamma = \frac{p}{\sqrt{(mn+mp+np)}}.$$

If, therefore, we construct a triangle, the radii of the escribed circles of which are equal to m, n, p, the half-angles of this triangle will be the req'd parts; for, denote the radii of the escribed circles by r_1, r_2, r_3, we have the relation $\tan \frac{1}{2}A = r_1 \div \sqrt{(r_1 r_2 + r_1 r_3 + r_2 r_3)}$.

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377. *By W. E. Heal.*—"If the equations,

$$x^2 + a\,x + b \ = 0$$
$$x^2 + a_1 x + b_1 = 0,$$

have a common root, find the remaining roots."

SOLUTION BY PROF. E. B. SEITZ.

Let m, r be the roots of the first equation, and n, r those of the second equation. Then, by the theory of equations, $m+r = -a$ (1), $mr = b$ (2), $n+r = -a_1$, (3), and $nr = b_1$ (4).

Dividing the diff. of (2) and (4) by the diff. of (1) and (3), we find

$$r = -\frac{b - b_1}{a - a_1}.$$

Substituting the value of r in (1) and (3), we find

$$m = \frac{b - b_1}{a - a_1} - a, \text{ and } n = \frac{b - b_1}{a - a_1} - a_1.$$

The condition that the equations have a common root is $(b - b_1)^2 = (a - a_1)(a_1 b - a b_1)$.

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378. No solution received. Prof. Casey says the proposition is not true.

379. *By Paul Peltier, A. M., Waterloo, Ill.*—"If any number of circles touch one another in one point, all their polars which correspond to a common pole, pass through a single point."

SOLUTION BY ALEXANDER ZIWET, DETROIT, MICH.

Take the common tangent of the circles for the axis of y, and the diameter to the point of contact for the axis of x; then the equation of any one of the circles is $x^2+y^2 = 2rx$; hence the equation of the polar is

$$x'x + y'y - r(x + x') = 0.$$

The lines represented by this equation for different values of r, i. e., the polars of the point (x', y') for the different circles, will all pass through the intersection of the lines $xx' + yy' = 0$, $x + x' = 0$, of which the former is perpendicular to the line joining (x', y') to the point of contact of the circles.

The problem is a special case of a more general problem, solved in Geo. Salmon's Conic Sections, Chapt. IX; as the common tangent may be regarded as the radical axis of the circles.

380. *By Lieut. Chas. A. Stone, U. S. Naval Acad., Ann., Md.*—"Find the equation of the curve in which the tangent of the angle which the tangent line makes with the axis of X, increases proportionally to the length of the curve."

SOLUTION BY PROF. DE VOLSON WOOD.

We have $\frac{dy}{dx} = as$; $\therefore\ \frac{d^2y}{dx^2} = a\sqrt{\left(1+\frac{dy^2}{dx^2}\right)}$, and

$$d\left(\frac{dy}{dx}\right) \div \sqrt{\left(1+\frac{dy^2}{dx^2}\right)} = ax.$$

Integrating

$$\log\left[\frac{dy}{dx} + \sqrt{\left(1+\frac{dy^2}{dx^2}\right)}\right] = ax + (C = 0).$$

Passing to exponentials, clearing of radicals, &c., and integrating, gives

$$y = \frac{1}{2a}\left(e^{ax} + e^{-ax}\right) + C,$$

which is the equation of the Catenary.

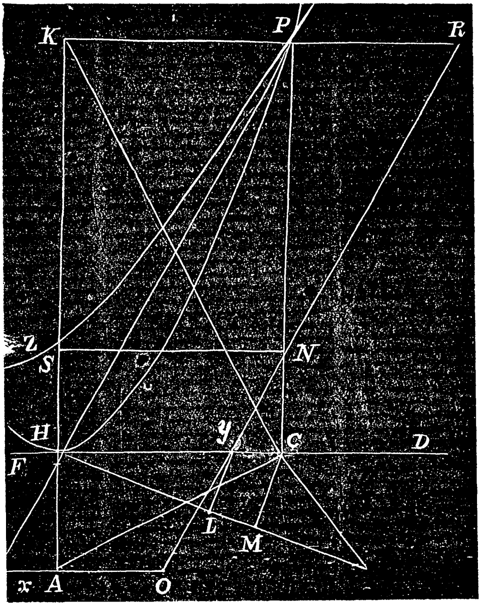
381. *By Prof. W. P. Casey.*—"To find a point in a given line so that the rectangle contained by two lines drawn to it from two given points may be given or a minimum (without the aid of the Cassinian Ovals)."

SOLUTION BY PROF. W. P. CASEY.

Let FD be the given line, A and B the given p'ts and C the req'd point.

Analysis.— Draw AH perp. to FB; $\therefore$ H is a given point. Join BH, it is a given line, and its middle point L, is a given point, and the perpend. LY to HB is in position; $\therefore$ Y is a given point. Draw CM perp. to BH and CK perp. to AC meeting AH produced in K, and make $AH \times HS = 2HB \times LM$, draw SN perp. to AK meeting the perp. CN to FD in the p't N; join NY and produce it to meet the perp's AO, KR to AK in O and R.

Now $CH^2 - CB^2 = HM^2 - MB^2 = 2HB \times LM = AH \times HS$; $HC^2 - AH \times HS = CB^2$, that is, $AH \times SK = CB^2$, $\therefore$ $AC^2.AH.SK = AC^2.CB^2$,



which is given, ∴ $AC^2\times AH\times SK$ is given and AH is a given line, ∴ $AC^2\times SK$ is given, or $AH\times AK\times SK$ is given, whence $AK\times SK$ is given; but as $AH\times HS=2HB\times LM$; ∴ $AH:2HB::LM:HS$ or CN a giv'n ratio, and $HL:HY::LM:YC$ a given ratio; ∴ the ratio of $NC:CY$ is given and the angle NCY is a right angle; whence the $\angle$s NYC, YNC are given, ∴ NY is in position, and the p't O is given. The ratio of $NP:PR$ is also given being the same as $AC:CY$, ∴ $AK\times NP:AK\times PR$ is given; but $AK\times NP$ or SK has been shown to be given ∴ $AK\times PR$ is given, or the parallelogram $XORP$ is given, hence the locus of P is an hyperbola PZ, whose asymptotes are OX, OR, and $AH\times HK=KP^2$, i. e., a given line multiplied by $HK=KP^2$, ∴ the locus of P is a parabola PH, hence P is a given point, and ∴ the perp. PC is in position and C is a given point. When the two curves become tang't there will be only one point from which a perp. to FB can be drawn, and in that case the product will be a minimum. There are two points when the curves intersect. The construction is obvious from the analyis.

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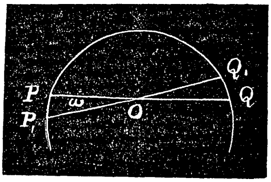
382. *By Thomas Spencer.*—"Prove in general that the chord drawn though a given point so as to cut off the minimum area from a given curve is bisected at that point."

SOLUTION BY ALX. S. CHRISTIE, U. S. COAST SURVEY.



Let P_1PQ_1Q be any given curve, PQ a chord through the given point O cutting off minimum area on one side of the chord, and, if the curve be closed, a maximum area on the other side.

Then taking P_1OQ_1 a consecutive position of the chord, ω infinitesimal, the areas OPP_1, OQQ_1 are, to terms of the 1st order, equal to $OP.OP_1.\omega$, $OQ.OQ_1.\omega$, respectively. But to the same order $OP=OP_1$, $OQ=OQ_1$; hence the element'y areas are $(OP)^2.\omega$, $(OQ)^2\times\omega$; and for either a maximum or a minimum these areas are equal, or

$$(OP)^2.\omega=(OQ)^2.\omega,\ \therefore\ OP=OQ.$$

383.—*By Prof. Edmunds.*—"Solve and discuss:

$$\begin{cases} x^2 \;+\; y^2 = a^2, \\ \log x + \log y = n.\text{''} \end{cases}$$

SOLUTION BY DR. A. B. NELSON.

Writing the second equation in the form $xy = e^n$, we readily find

$$x = \tfrac{1}{2}[\pm\surd(a^2+2e^n)\pm\surd(a^2-2e^n)],$$
$$y = \tfrac{1}{2}[\pm\surd(a^2+2e^n)\mp\surd(a^2-2e^n)].$$

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384. *By Prof. Asaph Hall.*—"Show that

$$\int_0^a dx \int_0^x \varphi(x, y)\,.\,dy = \int_0^a dy \int_y^a \varphi(x, y)dx.$$

(Dirichlet's theorem.)"

SOLUTION BY PROF. H. T. EDDY.

Consider the isosceles right angled triangle enclosed by the lines

$$x = y, \; x = a, \; y = 0.$$

The first member of the given equation extends the integration over the area of this triangle by finding first an elementary strip parallel to y betw'n $y = 0$ and $y = x$, and, secondly, taking sum of such strips between $x = 0$ and $x = a$.

The second member proposes to take first an elementary strip parallel to x between $x = y$ and $x = a$, and then sum all such strips between $y = 0$ and $y = a$. These summations are evidently equal.

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385. *Selected by Prof. H. T. Eddy.*—"Show that

$$\int_{-\infty}^{+\infty} \varepsilon^{\left(-x^2\cos 2\theta + \frac{a^2}{2x^2}\sin 2\theta\right)} \begin{matrix}\cos\\ \sin\end{matrix}\left[x^2 \sin 2\theta + \frac{a^2}{2x^2}\cos 2\theta\right]dx$$
$$= \pi^{\frac{1}{2}}\varepsilon^{-a} \begin{matrix}\cos\\ \sin\end{matrix}\left[\theta + a\right].\text{''}$$

SOLUTION BY HENRY HEATON.

We have given the definite integral

$$\int_{-\infty}^{+\infty} e^{-(y^2+c^2y^2)}dy = \surd\pi.e^{-2c}. \quad \text{(See Todhunter's Int. Calc. Art. 286.)}$$

Put $y = x(\cos\theta + i\sin\theta)$ and $c = \frac{1}{2}a(1+i)$, where $i = \surd-1$. Then is

$$\int_{-\infty}^{+\infty} e^{-[x^2\cos 2\theta + x^2 i \sin 2\theta + (a^2 i \div 2x^2)\cos 2\theta + (a^2 \div 2x^2)\sin 2\theta]}dx$$
$$= \surd\pi.e^{-a}e^{-ia}(\cos\theta - i\sin\theta).$$

But $e^{-i[x^2 \sin 2\theta + (a^2 \div 2x^2) \cos \theta]} = \cos [x^2 \sin 2\theta + (a^2 \div 2x^2) \cos 2\theta]$
$- i \sin [x^2 \sin 2\theta + (a^2 \div 2x^2) \cos 2\theta$, and
$e^{-ia} = \cos a - i \sin a$. Hence

$$\int_{-\infty}^{+\infty} e^{-[x^2 \cos 2\theta + (a^2 \div 2x^2) \sin 2\theta]} \left\{ \cos\left(x^2 \sin 2\theta + \frac{a^2}{2x^2} \cos 2\theta\right) - i \sin\left(x^2 \sin 2\theta + \frac{a^2}{2x^2} \cos 2\theta\right) \right\} dx = \sqrt{\pi}.e^{-a}[(\cos\theta \cos a - \sin\theta \sin a) - i (\cos\theta \sin a + \sin\theta \cos a)].$$

Separating the possible and impos. p'ts we have

$$\int_{-\infty}^{+\infty} e^{-[x^2 \cos 2\theta + (a^2 \div 2x^2) \sin 2\theta]} \cos\left(x^2 \sin 2\theta + \frac{a^2}{2x^2} \cos 2\theta\right) dx = \pi^{\frac{1}{2}} e^{-a} \cos(\theta + a),$$

$$\int_{-\infty}^{+\infty} e^{-[x^2 \cos 2\theta + (a^2 \div 2x^2) \sin 2\theta]} \sin\left(x^2 \sin 2\theta + \frac{a^2}{2x^2} \cos 2\theta\right) dx = \pi^{\frac{1}{2}} e^{-a} \sin(\theta + a).$$

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386. *By George Eastwood.*—"Integrate the equation

$$\frac{d^2\psi}{dt^2} \cdot \frac{d^2\psi}{dx^2} - \left(\frac{d\psi}{dt} \cdot \frac{d\psi}{dx}\right)^2 = 0."$$

SOLUTION BY PROF. L. G. BARBOUR.

Let $\frac{d\psi}{dt} = p$, $\frac{d\psi}{dx} = q$; then is $\frac{dp}{dt} \cdot \frac{dq}{dx} = p^2 q^2$. It is allowable and suff. to write $\frac{dp}{dt} = p^2$, and $\frac{dq}{dx} = q^2$, $\therefore \frac{dp}{p^2} = dt$; $-\frac{1}{p} = t + c$; $d\psi = -\frac{dt}{t+c}$
$\psi = -\log(t + c) + \log c'$, $\therefore e\phi = c' \div (t + c) = c''' \div (x + c''$, $\therefore x = tc_4 + c_5$.

PROBLEMS.

387. *By Prof. L. G. Barbour.*—Given the length of each side of any quadrilateral, and the distance from the middle point of any side to that of the side opposite. Required the distance from the middle point of one of the other sides to that of the side opposite.

388. *By Prof M. L. Comstock.*—F and F' being the foci of an ellipse and P a point on the curve, FD is drawn perpendic'r to FP meeting $F'P$ in D. Find the locus of D: (1) when $b > c$, (2) when $b = c$, (3) when $b < c$, if b = semi-minor axis and c = distance from the centre to either focus.